Preface

This book is an introduction to the mathematical concepts and methods used in economics. We assume that the reader has studied arithmetic and elementary algebra at school, and has some previous acquaintance with graphs; apart from that, the book is self-contained. In the course of our thirty-two chapters we cover all the calculus, matrix algebra, static and dynamic optimisation and probability theory that is required for undergraduate programmes in economics, and much of the mathematics needed for Master's programmes as well.

Our approach is to teach the student of economics how to *do* mathematics and how to *use* it sensibly. We emphasise the reasons why results are true and how they are related. We promote active learning with plentiful exercises and problems: rather than merely describe the use of mathematics in economics, we have included numerous problems where the student is asked to practise just this. Answers to all exercises and problems are available online; see page xvii for details.

We intend this book to be introductory in both senses of the word: we assume little in the way of prerequisites, but also provide a sound preparation for those who wish to proceed further. Combining these two attributes means more than just writing a long book. First, it means paying attention to pace, achieving momentum without breathlessness. For this reason we have not skimped on the very elementary material in the early chapters, which provide the basis on which the student can cope with a clean presentation of more advanced topics. Secondly, we have been very careful in matters of organisation; examples are noted below.

A third aspect of our determination to be introductory in both senses is our attitude to proof. We have attempted wherever possible to provide some explanation of what follows from what, giving non-rigorous (but not, we hope, misleading) plausibility arguments for results whose precise proofs are beyond our scope. We discourage rote-learning and try to avoid magic formulae. At the same time, we have deliberately adopted a discursive style and do not burden the reader with the definition-theorem-proof format. Our terminology follows a similar middle way. We define our terms, and strive to avoid serious ambiguity. On the other hand, we consider a certain laxity of language appropriate in a book of this nature; the use of 'the function f', 'the function $f(x)$' and 'the function $y = f(x)$' in close proximity to mean the same thing should not embarrass writer or reader. In the last two chapters, where we move up a gear in terms of rigour, we refine our language accordingly.

Two remarks on what this book is not. First, it is not about mathematics for its own sake: our selection of topics, examples and problems is driven by the needs of the economist. Secondly, this is not a textbook on 'economics done mathematically'. Many years ago, such books were a useful antidote to the then-dominant style of undergraduate economics. Today, all the best textbooks on intermediate microeconomics and macroeconomics are written by authors well versed in mathematical model-building, and adopt and encourage the model-builder's attitude to the subject. There is therefore no need for us to provide lengthy discussions of the relevant economics. But for a thorough understanding of modern economics attitude is not enough: students also need mathematical knowledge, technique and self-confidence. These needs become stronger as they progress to more advanced material.

Our aim, then, is to teach students of economics the mathematical skills they require, leaving the economics itself to be done elsewhere. We provide applications to many branches of economics in the text, in worked examples and especially in the exercises and problems. But the emphasis throughout is on helping the reader to master the mathematics.

Special features

The following list is a mixture of topics we emphasise more strongly than other books, things we believe we do particularly well and assorted idiosyncrasies and obsessions that have found their way into our book after many years of teaching. It is not intended to be exhaustive in any respect, but should give a flavour of what is to come.

- The material on linear programming in Chapter 2 has very little direct connection with anything else that happens before Chapter 18. It is, however, a wonderfully simple illustration of how to combine algebra with diagrams, a skill that is essential for understanding calculus.

- Our treatment of differentiation consistently emphasises links with linear approximations.

- Starting in Chapter 8, we provide a very full discussion of how traditional calculus needs to be tailored to economic optimisation problems. We emphasise the distinction between global and local optima, non-negativity constraints and the relevance of concavity and convexity.

- We adopt what is sometimes called a 'late transcendentals' approach to calculus. The exponential function is introduced in Chapter 9, after we have given a thorough account of the rules of differentiation and maxima and minima. As is natural in a book designed for economists, the concept of compound interest is used to motivate the exponential function and derive its properties; the power series comes later, in Chapter 10. Trigonometric functions and complex numbers are introduced later still, in Chapters 24 and 25, and are motivated by the need to model oscillatory behaviour.

- Our treatment of matrix algebra in Chapters 11 and 12 has two main themes: first, systems of linear equations and the use of row operations as the main tool for solving them; secondly, the interpretation of matrices as mappings. Only after this, in Chapter 13, do we come on to determinants and quadratic forms. We treat the former briefly and the latter in rather more detail, because of their importance for optimisation and for econometrics.

- The chapters on differential calculus of several variables follow those on matrix algebra. There is a very good reason for this: use of vectors and matrices makes it much easier to explain n–variable results by single-variable analogies. The gain is substantial even when $n = 2$.

- Chapter 15 contains a full and systematic treatment of comparative statics.

- Eigenvalues and eigenvectors appear very late, in Chapter 27: our initial discussion of quadratic forms in Chapter 13 does not require them. Our treatment of eigenvalue theory is elementary but thorough; we explain all the results needed for the following chapter on dynamic systems, and also include material much used in econometrics.

- In our discussion of dynamics and dynamic optimisation in Chapters 23, 26 and 28–30, we explain carefully both the similarities and differences between discrete-time and continuous-time approaches. This theme starts as early as Section 9.3 in the context of growth rates and interest rates.

- As we noted above, it is not our intention to duplicate material taught in economics textbooks. We have however paid considerable attention to interstitial topics, which tend to be omitted from mathematics courses because they are economics and vice versa. Examples are the already-mentioned Section 9.3 on time in economics, the use of phase diagrams in Chapter 28 and much of Chapter 18 on advanced topics in constrained optimisation; this chapter gives a thorough account of envelope theorems and the Kuhn–Tucker theorem.

- Chapters 21 and 22 on probability theory are placed immediately after the two chapters on integration, as they contain important applications of integral calculus. We explain carefully both the similarities and the differences between discrete and continuous random variables, in a manner similar to our treatment of discrete and continuous time in the chapters on dynamics. Section 22.3 on bivariate distributions emphasises conditional expectation rather than joint density functions; we think that such a treatment is both cleaner and more relevant to economic applications than more conventional ones.

- The last two chapters of the book provide a bridge between calculus and the rigorous mathematical analysis used in graduate-level economic theory. The most immediate payoff from a rigorous approach is an insight into the circumstances in which problems have solutions. Our brief introduction to analysis emphasises this aspect of the subject.

Prerequisites and pathways

The minimal prerequisites for reading this book are some competence at algebraic manipulation and familiarity with graphs. At the same time, there is plenty in the book for students who have already studied some calculus. As a rough guide to the amount of time required to get through the book in typical undergraduate courses, we suggest that most of Chapters 1–20 and either 21 or 23 can be covered in two semesters by students without calculus, or in one semester by students with a good grounding in one-variable calculus. A further semester would be needed for the remainder of Chapters 21–28 and either Chapters 29 and 30 or 31 and 32.

Here are some suggestions for courses based on the book. A *first course for economics students with no previous knowledge of calculus* would consist of Chapters 1 to 9 and 19, possibly with selected material from Chapters 10, 20 and 21. A *second course, covering elementary matrix algebra, functions of several variables and some simple dynamics*, could start with a review of Sections 1.3 and 1.4 and then go through Chapters 11–17 and 23; some material from Chapter 18 could substitute for the harder comparative statics in Chapter 15. A course on *further calculus for economists* for students who are competent in one-variable calculus could start with a review of Chapters 8 and 9 with emphasis on the economic applications, give a brief account of partial differentiation using selected material from Chapter 14, review Chapters 19 and 20 with emphasis on economic applications, and then proceed through Chapters 23–26. A course on *optimisation, probability and dynamics for advanced undergraduates* could be based on Chapters 16–18, 21–28 and possibly 29 and 30; a course for *beginning graduate students* would add Chapters 31 and 32. There are numerous other possibilities; the table on page xvi should give a good idea of what variations in order are feasible.

Style

Chapters are of course numbered, as are sections within chapters. When we number propositions or worked examples, the numbering starts afresh within each section. Equation numbering, by contrast, is by chapter rather than section. The object of these conventions is to avoid intimidating the reader with a surfeit of dots.

Equations are numbered for future reference and for no other reason. Numbered equations should not be regarded as more important than unnumbered ones.

This book contains several hundred exercises and 128 problems (four per chapter). Exercises are at the end of sections and are relatively undemanding. Problems are at the end of chapters: they are often related to more than one section of the chapter, and some are quite hard. As noted above, answers are available online; see page xvii

Some chapters are followed by appendices which take up particular points, largely in order to add a little more rigour to the argument. Knowledge of the material in the appendices is not necessary for doing the exercises or problems, which is why we have put the appendices after the problems.

On nice points of mathematical terminology we have followed E. J. Borowski and J. M. Borwein's *Collins Dictionary of Mathematics*, Second Edition (Collins, 2002).

New in the fourth edition

The main innovation is the two chapters on probability theory. Some knowledge of probability is part of the toolkit of every economics student, not only as a background for econometrics but also in economic theory, both micro and macro. We think that probability is currently taught to economics undergraduates, at least in the UK, in a rather unsatisfactory way: the basic notions are covered in an almost maths-free first-year statistics course and the techniques needed for subsequent applications are introduced in a hurried and ad-hoc fashion. Something similar happens at the Master's level, where explanations tend to be even more hurried. Our view is that the mathematical principles of probability should be incorporated in the maths sequence, and our new chapters demonstrate how this may be done. They also provide a helpful service under current curricular practice: the lecturer charged with teaching applications of probability to students with an inadequate preparation in the theory needs to point them to a reference which will enable them to fill the gaps, and our chapters do exactly this.

Other new features include the following:

- A new section on limits and continuity in Chapter 5 eases the transition to differential calculus in the following chapter.
- Geometrical aspects of linear algebra receive more attention than in earlier editions.
- We have made several changes to Chapters 16–18 on static optimisation, clarifying the explanation of saddle points in Chapter 16 and providing more detail on sufficient conditions for an optimum in the following two chapters.
- The section introducing complex numbers has been expanded, so as to make the concept appear less mysterious.
- All diagrams have again been thoroughly revised.

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